System Level Performance Analysis - the SymTA/S Approach

Rafik Henia, Arne Hamann, Marek Jersak, Razvan Racu, Kai Richter, Rolf Ernst
Institute of Computer and Communication Network Engineering
Technical University of Braunschweig
D-38106 Braunschweig / Germany
{henia|hamann|jersak|racu|richter|ernst}@ida.ing.tu-bs.de

Abstract

SymTA/S is a system-level performance and timing analysis approach based on formal scheduling analysis techniques and symbolic simulation. The tool supports heterogeneous architectures, complex task dependencies and context aware analysis. It determines system-level performance data such as end-to-end latencies, bus and processor utilization, and worst-case scheduling scenarios. SymTA/S furthermore combines optimization algorithms with system sensitivity analysis for rapid design space exploration. This paper gives an overview of the current research interests in the SymTA/S project.

1. Introduction

With increasing embedded system complexity, there is a trend towards heterogeneous, distributed architectures. Multiprocessor system on chip designs (MpSoCs) use complex on-chip networks to integrate multiple programmable processor cores, specialized memories, and other intellectual property (IP) components on a single chip. MpSoCs have become the architecture of choice in industries such as network processing, consumer electronics, and automotive systems. Their heterogeneity inevitably increases with IP integration and component specialization, which designers use to optimize performance at low power consumption and competitive cost. Tomorrow’s MpSoCs will be even more complex, and using IP library elements in a 'cut-and-paste' design style is the only way to reach the necessary design productivity.

Systems integration is becoming the major challenge in MpSoC design. Embedded software is increasingly important to reach the required productivity and flexibility. The complex hardware and software component interactions pose a serious threat to all kinds of performance pitfalls, including transient overloads, memory overflow, data loss, and missed deadlines. The International Technology Roadmap for Semiconductors, 2003 Edition, (http://public.itrs.net/Files/2003ITRS/Design2003.pdf) names system-level performance verification as one of the top three codesign issues.
Simulation is state of the art in MpSoC performance verification. Tools from many suppliers support cycle-accurate cosimulation of a complete hardware and software system. The cosimulation times are extensive, but developers can use the same simulation environment, simulation patterns, and benchmarks in both function and performance verification. Simulation-based performance verification, however, has conceptual disadvantages that become disabling as complexity increases.

MpSoC hardware and software component integration involves resource sharing that is based on operating systems and network protocols. Resource sharing results in a confusing variety of performance runtime dependencies. For example, figure 1 shows a CPU subsystem executing three processes. Although the operating system activates $T_1$, $T_2$, and $T_3$ strictly periodically (with periods $P_1$, $P_2$, and $P_3$, respectively), the resulting execution sequence is complex and leads to output bursts.

As figure 1 shows, $T_1$ can delay several executions of $T_3$. After $T_1$ completes, $T_3$—with its input buffers filled—temporarily runs in burst mode with the execution frequency limited only by the available processor performance. This leads to transient $T_3$ output burst, which is modulated by $T_1$’s execution.

Figure 1 does not even include data-dependent process execution times, which are typical for software systems, and operating system overhead is neglected. Both effects further complicate the problem. Yet finding simulation patterns—or use cases—that lead to worst-case situations as highlighted in Figure 1 is already challenging.

Network arbitration introduces additional performance dependencies. Figure 2 shows an example. The arrows indicate performance dependencies between the CPU and DSP subsystems that the system function does not reflect. These dependencies can turn component or subsystem best-case performance into system worst-case performance—a so-called scheduling anomaly. Recall the $T_3$ bursts from Figure 1 and consider that $T_3$’s execution time can vary from one execution to the next.

There are two critical execution scenarios, called corner cases: The minimum execution time for $T_3$ corresponds to the maximum transient bus load, slowing down other components’ communication, and vice versa.

The transient runtime effects shown in figures 1 and 2 lead to complex system-level corner cases. The designer must provide a simulation pattern that reaches each corner case during simulation. Essentially, if all corner cases satisfy the given performance constraints, then the system is guaranteed to satisfy its constraints under all possible operation conditions.
ever, such corner cases are extremely difficult to find and debug, and it is even more difficult to find simulation patterns to cover them all. Reusing function verification patterns is not sufficient because they do not cover the complex nonfunctional performance dependencies that resource sharing introduces. Reusing component and subsystem verification patterns is not sufficient because they do not consider the complex component and subsystem interactions.

The system integrator might be able to develop additional simulation patterns, but only for simple systems in which the component behavior is well understood. Manual corner case identification and pattern selection is not practical for complex MpSoCs with layered software architectures, dynamic bus protocols, and operating systems. In short, simulation-based approaches to MpSoC performance verification are about to run out of steam, and should essentially be enhanced by formal techniques that systematically reveal and cover corner cases.

Real-time systems research has addressed scheduling analysis for processors and buses for decades, and many popular scheduling analysis techniques are available. Examples include rate-monotonic scheduling and earliest deadline first [16], using both static and dynamic priorities; and time-slicing mechanisms like TDMA or round-robin [4]. Some extensions have already found their way into commercial analysis tools, which are being established e.g. in the automotive industry to analyze individual units that control the engine or parts of the electronic stability program.

The techniques rely on a simple yet powerful abstraction of task activation and communication. Instead of considering each event individually, as simulation does, formal scheduling analysis abstracts from individual events to event streams. The analysis requires only a few simple characteristics of event streams, such as an event period or a maximum jitter. From these parameters, the analysis systematically derives worst-case scheduling scenarios, and timing equations safely bound the worst-case process or communication response times.

It might surprise that –up to now– only very few of these approaches have found their way into the SoC (system-on-chip) design community by means of tools. Regardless of the known limitations of simulation such as incomplete corner-case coverage and pattern generation, timed simulation is still the preferred means of performance verification in MpSoC design. Why then is the acceptance of formal analysis still very limited?

One of the key reasons is a mismatch between the scheduling models assumed in most formal analysis approaches and the heterogenous world of MpSoC scheduling techniques and communication patterns that are a result of a) different application
characteristics; b) system optimization and integration which is still at the beginning of the MpSoC development towards even more complex architectures.

Therefore, a new configurable analysis process is needed that can easily be adapted to such heterogeneous architectures. We can identify different approaches: the holistic approach that searches for techniques spanning several scheduling domains; and hierarchical approaches that integrate local analysis with a global flow based analysis, either using new models or based on existing models and analysis techniques.

In the following section, the existing analysis approaches from the literature on real-time analysis are deeply reviewed and key requirements for their application to MpSoC design are identified. In Section 3, the fundamentals and basic models of the SymTA/S technology are introduced. Section 4 surveys a large number of extensions that enable the analysis of complex applications. Section 5 shows how the overall analysis accuracy can be deliberately increased when designers specify few additional correlation information. Automatic optimizations using evolutionary algorithms is explained in Section 6, while Section 7 introduces the idea of sensitivity analysis. An experiment is carried out in section 8. Experimental results are interpreted, before drawing conclusions.

2. Formal Techniques in System Performance Analysis

Formal approaches to heterogeneous systems are rare. The “holistic” approach [28, 6] systematically extends the classical scheduling theory to distributed systems. However, because of the very large number of dependencies, the complexity of the equations underlying the analysis grows with system size and heterogeneity. In practice, the holistic approach is limited to those system configurations which simplify the equations, such as deterministic TDMA networks. However, there is, up to now, no general procedure to set-up and solve the holistic equations for arbitrary systems. This could explain, why such holistic approaches are largely ignored by the SoC community even though there are many proposals for multiprocessor analysis in real-time computing.

Gresser [5] and Thiele [27] established a different view on scheduling analysis. The individual components or subsystems are seen as entities which interact, or communicate, via event streams. Mathematically speaking, the stream representations are used to capture the dependencies between the equations (or equations sets) that describe the individual components timing. The difference to the holistic approach (that also captures the timing using system-level equations) is that the compositional models are well-structured with respect to the architecture. This is considered a key benefit, since the structuring significantly helps designers to understand the complex dependencies in the system, and it enables a surprisingly simple solution. In the “compositional” approach, an output event stream of one component turns into an input event stream of a connected component. Schedulability analysis, then, can be seen as a flow-analysis problem for event streams that, in principle, can be solved iteratively using event stream propagation.

Both approaches use a highly generalized event stream representation to tame the complexity of the event streams. Gresser uses a superpositional event vector system, which is then propagated using complex event dependency matrices. Thiele et.al. use a more intuitive model. They use numerical upper and lower bound event arrival curves for event streams, and similar service curves for execution modeling.

This generality, however, has its price. Because they introduced new stream models, both Thiele and Gresser had to develop
new scheduling analysis algorithms for the local components that utilize these models; the host of existing work in real-time system can not be re-used. Furthermore, the new models are far less intuitive than the ones known from the classical real-time systems research, e. g. the model of rate-monotonic scheduling with its periodic tasks and worst-case execution times. A system-level analysis should be simple and comprehensible, otherwise its acceptance is extremely doubtful.

The compositional idea is a good starting point for the following considerations. It uses some event stream representation to allow component-wise local analysis. The local analysis results are, then, propagated through the system to reach a global analysis result. We don’t necessarily need to develop new local analysis techniques if we can benefit from the host of work in real-time scheduling analysis. For example, in Figure 1, even if input and output streams seem to have totally different characteristics, the number of $T_3$’s output events can be easily bounded over a longer time interval. The bursts only occur temporarily, representing a transient overload within a generally periodic event stream. In other words, some key characteristics of the original periodic stream remain even in the presence of heavy distortion.

A key novelty of the SymTA/S approach is that it uses intuitive standard event models (section 3.2) from real-time systems research rather than introducing new, complex stream representations. Periodic events or event streams with jitter and bursts [31] are examples of standard models that can be found in literature. The SymTA/S technology allows to extract this information from a given schedule and automatically interface or adapt the event stream to the specific needs within these standard models, so that designers and analysts can safely apply existing subsystem techniques of choice without compromising global analysis.

3. The SymTA/S approach

SymTA/S [7] is a formal system-level performance and timing analysis tool for heterogeneous SoCs and distributed systems. The application model of SymTA/S is described in section 3.1. The core of SymTA/S is a technique to couple local scheduling analysis algorithms using event streams [21, 24]. Event streams describe the possible I/O timing of tasks. Input and output event streams are described by standard event models which are introduced in detail in section 3.2. The analysis composition using event streams is described in section 3.3. A second key property of the SymTA/S compositional approach is the ability to adapt the possible timing of events in an event stream. The event stream adaptation concept is described in section 3.4.

3.1 SymTA/S application model

A task is activated due to an activating event. Activating events can be generated in a multitude of ways, including expiration of a timer, external or internal interrupt, and task chaining. Each task is assumed to have one input FIFO. A task reads its activating data from its input FIFO and writes data into the input FIFO of a dependent task. A task may read its input data at any time during one execution. The data is therefore assumed to be available at the input during the whole execution of the task. SymTA/S also assumes that input data is removed from the input FIFO at the end of one execution.

A task needs to be mapped on a computation or communication resource to execute. When multiple tasks share the same resource, then two or more tasks may request the resource at the same time. In order to arbitrate request conflicts, a resource is associated with a scheduler which selects a task to which it grants the resource out of the set of active tasks according to some scheduling policy. Other active tasks have to wait. Scheduling analysis calculates worst-case (sometimes also best-
case) task response times, i.e. the time between task activation and task completion, for all tasks sharing a resource under
the control of a scheduler. Scheduling analysis guarantees that all observable response times will fall into the calculated
[best-case, worst-case] interval. Scheduling analysis is therefore conservative. A task is assumed to write its output data at
the end of one execution. This assumption is standard in scheduling analysis.

Figure 3. System modeled with SymTA/S

Figure 3 shows an example of a system modeled with SymTA/S. The system consists of 2 resources each with 2 tasks mapped
on it. R1 and R2 are both assumed to be priority scheduled. Src1 and Src2 are the sources of the external activating events
at the system inputs. The possible timing of activating events is captured by so-called event models, which are introduced in
section 3.2.

3.2 SymTA/S standard event models

Event models can be described by sets of parameters. For example, a periodic with jitter event model has two parameters
(\(P, J\)) and states that each event generally occurs periodically with period \(P\), but that it can jitter around its exact position
within a jitter interval \(J\). Consider an example where \((P, J) = (4, 1)\). This event model is visualized in figure 4. Each
gray box indicates a jitter interval of length \(J = 1\). The jitter intervals repeat with the event model period \(P = 4\). The
figure additionally shows a sequence of events which satisfies the event model, since exactly one event falls within each jitter
interval box, and no events occur outside the boxes.

Figure 4. Example of an event stream that satisfies the event model \((P = 4, J = 1)\)

An event model can also be expressed using two event functions \(\eta^{u}(\Delta t)\) and \(\eta^{l}(\Delta t)\).
**Definition 1 (Upper Event Function)** The upper event function \( \eta^u(\Delta t) \) specifies the maximum number of events that can occur during any time interval of length \( \Delta t \).

**Definition 2 (Lower Event Function)** The lower event function \( \eta^l(\Delta t) \) specifies the minimum number of events that have to occur during any time interval of length \( \Delta t \).

Event functions are piecewise constant step functions with unit-height steps, each step corresponding to the occurrence of one event. Figure 5 shows the event functions for the event model \((P = 4, J = 1)\). Note that at the points where the functions step, the smaller value is valid for the upper event function, while the larger value is valid for the lower event function (indicated by dark dots). For any time interval of length \( \Delta t \), the actual number of events is bound by the upper and lower event functions.

Event functions resemble arrival curves [3] which have been successfully used by Thiele et al. for compositional performance analysis of network processors [26]. In the following, the dependency of \( \eta^u \) and \( \eta^l \) on \( \Delta t \) is omitted for brevity.

![Figure 5. Upper and lower event functions for the event model \((P = 4, J = 1)\)](image)

A periodic with jitter event model is described by the following event functions \( \eta^u_{P+J} \) and \( \eta^l_{P+J} \) [23].

\[
\eta^u_{P+J} = \left\lfloor \frac{\Delta t + J}{P} \right\rfloor \tag{1}
\]

\[
\eta^l_{P+J} = \max \left(0, \left\lfloor \frac{\Delta t - J}{P} \right\rfloor \right) \tag{2}
\]

To get a better feeling for event functions, imagine a sliding window of length \( \Delta t \) that is moved over the (infinite) length of an event stream. Consider \( \Delta t = 4 \) (gray vertical bar in figure 5). The upper event function indicates that at most 2 events can be observed during any time interval of length \( \Delta t = 4 \). This corresponds e.g. to a window position between \( t_0 + 8.5 \) and \( t_0 + 12.5 \) in figure 4. The lower event function indicates that no events have to be observed during \( \Delta t = 4 \). This corresponds e.g. to a window position between \( t_0 + 12.5 \) and \( t_0 + 16.5 \) in figure 4.

In addition, distance functions \( \delta_{\min}(N \geq 2) \) and \( \delta_{\max}(N \geq 2) \), are defined to return the minimum respectively maximum distance between \( N \geq 2 \) consecutive events in an event stream.
Definition 3 (Minimum Distance Function) The minimum distance function $\delta_{\text{min}}(N \geq 2)$ specifies the minimum distance between $N \geq 2$ consecutive events in an event stream.

Definition 4 (Maximum Distance Function) The maximum distance function $\delta_{\text{max}}(N \geq 2)$ specifies the maximum distance between $N \geq 2$ consecutive events in an event stream.

For periodic with jitter event models the following distance functions are obtained

\[ \delta_{\text{min}}(N \geq 2) = \max\{0, (N - 1) \cdot P - J\} \]  
\[ \delta_{\text{max}}(N \geq 2) = (N - 1) \cdot P + J \]

For example, the minimum distance between 2 events in a periodic with jitter event model with $(P = 4, J = 1)$ is 3 time units, and the maximum distance between 2 events is 5 time units.

Periodic with jitter event models are well suited to describe generally periodic event streams, which often occur in control, communication and multimedia systems [22]. If the jitter is zero, then the event model is strictly periodic. If the jitter is larger than the period, then two or more events can occur at the same time, leading to bursts. To describe a bursty event model, the periodic with jitter event model can be extended with a $d_{\text{min}}$ parameter that captures the minimum distance between events within a burst. A more detailed discussion can be found in [23].

Additionally, sporadic events are also common [22]. Sporadic event streams are modeled with the same set of parameters as periodic event streams. The difference is that for sporadic event streams, the lower event function $\eta(\Delta t)$ is always zero. The maximum distance function $\delta_{\text{max}}(N \geq 2)$ approaches infinity for all values of $N$ [23]. Note that jitter and $d_{\text{min}}$ parameters are also meaningful in sporadic event models, since they allows to accurately capture sporadic transient load peaks.

Event models with this small set of parameters have several advantages. Firstly, they are easily understood by a designer, since period, jitter etc. are familiar event stream properties. Secondly, the corresponding event functions and distance functions can be evaluated quickly, which is important for scheduling analysis to run fast. Thirdly, as will be shown in section 3.3.2, compositional performance analysis requires the modeling of possible timing of output events for propagation to the next scheduling component. Event models as described above allow to specify simple rules to obtain output event models (section 3.3.1) that can be described with the same set of parameters as the activating event models. Therefore, there is no need to depart from this event models independent of size and structure of the composed system (hence the term ‘standard’). This makes the compositional performance analysis approach very general.

3.3 Analysis composition

In the compositional performance analysis methodology [22, 23], local scheduling analysis and event model propagation are alternated, during system-level analysis. This requires the modeling of possible timing of output events for propagation to the next scheduling component. In the following, first the output event model calculation is explained. Then the compositional analysis approach is presented.
3.3.1 Output event model calculation

The SymTA/S standard event models allow to specify simple rules to obtain output event models that can be described with the same set of parameters as the activating event models. The output event model period obviously equals the activation period. The difference between maximum and minimum response times (the response time jitter) is added to the activating event model jitter, yielding the output event model jitter (equation 5).

\[ J_{\text{out}} = J_{\text{act}} + (t_{\text{resp,max}} - t_{\text{resp,min}}) \] (5)

Note that if the calculated output event model has a larger jitter than period, this information alone would indicate that an early output event could occur before a late previous output event, which obviously cannot be correct. In reality, output events cannot follow closer than the minimum response time of the producer task. This is indicated by the value of the minimum distance parameter.

3.3.2 Analysis composition using standard event models

In the following, the compositional analysis approach is explained using the system example in figure 3. Initially, only event models at the external system inputs are known. Since an activating event model is available for each task on \( R_1 \), a local scheduling analysis of this resource can be performed and output event models are calculated for \( T_1 \) and \( T_3 \) (section 3.3.1). In the second phase, all output event models are propagated. The output event models become the activating event models for \( T_2 \) and \( T_4 \). Now, a local scheduling analysis of \( R_2 \) can be performed since all activating event models are known.

However, it is sometimes impossible to perform system level scheduling analysis as explained above. This is shown in the system example in figure 6.

Figure 6. Example of a system with cyclic scheduling dependency

Figure 6 shows a system consisting of 2 resources, \( R_1 \) and \( R_2 \), each with 2 tasks mapped on it. Initially, only the activating event models of \( T_1 \) and \( T_3 \) are known. At this point the system cannot be analyzed, because on every resource an activating event model for one task is missing. I.e. response times on \( R_1 \) need to be calculated to be able to analyze \( R_2 \). On the other hand, \( R_1 \) cannot be analyzed before analyzing \( R_2 \). This problem is called cyclic scheduling dependency.

One solution to this problem is to initially propagate all external event models along all system paths until an initial activating event model is available for each task [20]. This approach is safe since on one hand scheduling cannot change an event model
period. On the other hand, scheduling can only increase an event model jitter [31]. Since a smaller jitter interval is contained in a larger jitter interval, the minimum initial jitter assumption is safe.

After propagating external event models, global system analysis can be performed. A global analysis step consists of two phases [23]. In the first phase local scheduling analysis is performed for each resource and output event models are calculated (section 3.3.1). In the second phase, all output event models are propagated. It is then checked if the first phase has to be repeated because some activating event models are no longer up-to-date, meaning that a newly propagated output event model is different from the output event models that was propagated in the previous global analysis step. Analysis completes if either all event models are up-to-date after the propagation phase, or if an abort condition, e.g. the violation of a timing constraint has been reached.

3.4 Event Stream Adaptation

A key property of the SymTA/S compositional performance analysis approach is the ability to adapt the possible timing of events in an event stream (expressed through the adaptation of an event model [23]). There are several reasons to do this. It may be that a scheduler or a scheduling analysis for a particular component requires certain event stream properties. For example, rate-monotonic scheduling and analysis [16] require strictly periodic task activation. Alternatively, an integrated IP component may require certain event stream properties. External system outputs may also impose event model constraints, e.g. a minimum distance between output events or a maximum acceptable jitter. Such a constraint may be the result of a performance contract with an external subsystem [29]. Event stream adaptation can also be done for the sole purpose of traffic shaping [23]. Traffic shaping can be used e.g. to reduce transient load peaks, in order to obtain more regular system behavior. Practically, event model adaptation is distinguished from event model shaping in SymTA/S [25]. Adaptation is required to satisfy an event model constraint, while shaping is voluntary to obtain more regular system behavior. Two types of event adaptation functions (EAF) are currently implemented in SymTA/S: a periodic EAF produces periodic event stream from a periodic with jitter input event stream. A $d_{\text{min}}$-EAF enforces a minimum distance between output events.

4. Complex embedded applications

Compositional performance analysis as described so far is not applicable to embedded applications with complex task dependencies. This is because it uses a simple activation model where the completion of one task directly leads to the activation of a dependent task. However, activation dependencies in realistic embedded applications are usually more complex. A consumer task may require a different amount of data per execution than produced by a producer task, leading to multi-rate systems. Task activation may also be conditional, leading to execution-rate intervals. Furthermore, a task may consume data from multiple task inputs. Then, task activation timing is a function of the possible arrival timing of all required input data. Task with multiple inputs also allow to form cyclic dependencies (e. g. in a control loop).

In this section, the focus is on multiple inputs (both AND- and OR-activation) and functional cycles [11]. Multi-rate systems and conditional communication are not considered, since these features have not yet been incorporated into SymTA/S. The reader interested in their theoretical foundations is referred to [10].
4.1 Basic thoughts

The activation function of a consumer task $C$ with multiple inputs is a boolean function of input events at the different task inputs. An imposed restriction is that activation must not be invalidated due to the arrival of additional tokens [34]. This means that negation is not allowed in the activation function. Consequently, the only acceptable boolean operators are AND and OR, since an input is negated in all other commonly used boolean operators (NOT, XOR, NAND, NOR).

In order to perform scheduling analysis on the resource to which task $C$ is mapped, activating event functions for task $C$ have to be calculated from all input event functions. In the following it is shown how to do this for AND- and OR-activation using standard event models (section 3.2). An extended discussion covering event models in general can be found in [10].

4.2 AND-activation

For a consumer task $C$ with multiple inputs, AND-activation implies that $C$ is activated after an input event has occurred at each input $i$. An example of an AND-activated task with three inputs is shown in figure 7.

![Figure 7. Example of an AND-activated task C](image)

Note that AND-activation requires input data buffering, since at some inputs data may have to wait until data has arrived at all other inputs for one consumer activation. The term AND-buffering is used to refer to this source of buffering. The term token [14] is used to refer to the amount of data required for one input event.

4.2.1 AND-Activation Period

To ensure bounded AND-buffer sizes, the period of all input event models must be the same. The period of the activating event model equals this period.

\[
\begin{align*}
\mathcal{P}_i & = \mathcal{P}_j ; \quad i, j = 1..k \\
\mathcal{P}_{AND} & = \mathcal{P}_i ; \quad i = 1..k
\end{align*}
\] (6)
In order to obtain the AND-activation jitter, consider how often activation of the AND-activated task can occur during any time interval $\Delta t$. Obviously, during any time interval $\Delta t$, the port with the smallest minimum number of available tokens determines the minimum number of AND-activations. Likewise, the port with the smallest maximum number of available tokens determines the maximum number of AND-activations.

The number of available tokens at port $i$ during a time interval $\Delta t$ depends on both the number of tokens arriving during $\Delta t$, and on the number of tokens that arrived earlier, but did not yet lead to an activation because tokens at one or more other ports are still missing. This is illustrated in the following example. Assume that the task in figure 7 receives tokens at each with the following periodic with jitter input event models:

\[
\begin{align*}
\mathcal{P}_1 &= 4, \quad J_1 = 0 \\
\mathcal{P}_2 &= 4, \quad J_2 = 2 \\
\mathcal{P}_3 &= 4, \quad J_3 = 3
\end{align*}
\]

Figure 8 shows a possible sequence of input events that adhere to these event models, and the resulting AND-activation events. The numbering of events in the figure indicates which events together lead to one activation of AND-activated task $C$.

As can be seen, the minimum distance between two AND-activations (activations 3 and 4 in figure 8) equals the minimum distance between two input events at input 3, which is the input with the largest input event model jitter. Likewise, the maximum distance between two AND-activations (activations 1 and 2 in figure 8) equals the maximum distance between two input events at input 3. It is not possible to find a different sequence of input events leading to a smaller minimum or a larger maximum distance between two AND-activations. From this it results that the input with the largest input event jitter determines the activation jitter of the AND-activated task. I. e.
\[ J_{AND} = \max\{J_i\} ; \quad i = 1..k \]  

(7)

This statement also remains true if the first set of input events do not arrive at the same time (as is the case in figure 8). A proof is given in [10]. Calculation of the worst-case delay and backlog at each input due to AND-buffering can also be found in [10].

Note that in some cases it may be possible to calculate phases between the arrival of corresponding tokens in more detail, e.g. through the use of inter-event-stream contexts (section 5.3). It may then be possible to calculate a tighter activating jitter if it can be shown that a certain input cannot (fully) influence the activation timing of an AND-activated task, because tokens at this input arrive relatively early. This is particularly important for the analysis of functional cycles (section 4.4).

4.3 OR-activation

For a consumer task \( C \) with multiple inputs, OR-activation implies that \( C \) is activated each time an input event occurs at any input of \( C \). Different to AND-activation, input event models are not restricted, and no OR-buffering is required, since a token at one input never has to wait for tokens to arrive at a different input in order to activate \( C \). Of course, activation buffering is still required.

![Figure 9. Example of an OR-activated task \( C \)](image)

An example of an OR-activated task with two inputs is shown in figure 9. Assume the following periodic with jitter event models at the two inputs of task \( C \):

\[ P_1 = 4, \quad J_1 = 2 \]
\[ P_2 = 3, \quad J_2 = 2 \]

The corresponding upper and lower input event functions are shown in figure 10. Since each input event immediately leads to one activation of task \( C \), the upper and lower activating event functions are constructed by adding the respective input event functions. The result is shown in figure 11(a).

Recall a key requirement of compositional performance analysis, namely that event streams are described in a form that can serve both as input for local scheduling analysis, and can be produced as an output of local scheduling analysis for propagation to the next analysis component (section 3.3.2). Due to the irregularly spaced steps (visible in figure 11(a)), the exact activating event functions cannot be described by a periodic with jitter event model, and thus cannot serve directly as
Figure 10. Upper and lower input event functions in the OR-example

Figure 11. Upper and lower activating event functions in the OR-example

input for local scheduling analysis. Furthermore, after local scheduling analysis a periodic with jitter output event model has to be propagated to the next analysis component. An activation jitter is required in order to calculate an output jitter (section 3.3.1). Therefore, conservative approximations for the exact activating event functions that can be described by a periodic with jitter event model (\[P_{OR}, J_{OR}\]) needs to be found. The intended result is shown in figure 11(b) (the exact curves appear as dotted lines).
4.3.1 OR-Activation Period

The period of OR-activation is the least common multiple LCM($P_i$) of all input event model periods (the *macro period*), divided by the sum of input events during the macro period assuming zero jitter for all input event streams.

\[
P_{OR} = \frac{\text{LCM}(P_i)}{\sum_{i=1}^{n} \frac{1}{P_i}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{P_i}} \tag{8}
\]

4.3.2 OR-Activation Jitter

A conservative approximation for the exact activating event functions with a periodic with jitter event model implies the following inequations.

\[
\left\lceil \frac{\Delta t + J_{OR}}{P_{OR}} \right\rceil \geq \sum_{i=1}^{n} \left\lceil \frac{\Delta t + J_i}{P_i} \right\rceil \tag{9}
\]

\[
\max\left(0, \left\lfloor \frac{\Delta t - J_{OR}}{P_{OR}} \right\rfloor \right) \leq \sum_{i=1}^{n} \max\left(0, \left\lfloor \frac{\Delta t - J_i}{P_i} \right\rfloor \right) \tag{10}
\]

In order to be as accurate as possible, the minimum jitter that satisfies inequations 9 and 10 must be found. It can be shown that the minimum jitter that satisfies inequation 9 and the minimum jitter that satisfies inequation 10 are the same [10]. In the following, the upper approximation (inequation 9) is used to calculate the OR-activation jitter. Since the left and right sides of this inequation are only piecewise continuous, the inequation cannot be simply transformed to obtain the desired minimum jitter. The solution used here is to evaluate inequation 9 piecewise for each interval $[\Delta t_j, \Delta t_{j+1}]$, during which the right side of the inequation has a constant value $k_j \in \mathbb{N}$. For each constant piece of the right side, a condition for a local jitter $J_{OR,j}$ is obtained that satisfies the inequation for all $\Delta t : \Delta t_j < \Delta t \leq \Delta t_{j+1}$.

For each constant piece of the right side, inequation 9 becomes

\[
\left\lceil \frac{\Delta t + J_{OR,j}}{P_{OR}} \right\rceil \geq k_j ; \quad \Delta t_j < \Delta t \leq \Delta t_{j+1}, k_j \in \mathbb{N}
\]

Since the left side of this inequation is monotonically increasing with $\Delta t$, it is sufficient to evaluate it for the smallest value of $\Delta t$, which approaches $\Delta t_j$. I. e.

\[
\lim_{\epsilon \to +0} \left\lceil \frac{\Delta t_j + \epsilon + J_{OR,j}}{P_{OR}} \right\rceil \geq k_j ; \quad k_j \in \mathbb{N}
\]

\[
\Leftrightarrow \lim_{\epsilon \to +0} \frac{\Delta t_j + \epsilon + J_{OR,j}}{P_{OR}} > k_j - 1
\]

\[
\Leftrightarrow \lim_{\epsilon \to +0} (J_{OR,j} + \epsilon) > (k_j - 1) * P_{OR} - \Delta t_j
\]

\[
\Leftrightarrow J_{OR,j} \geq (k_j - 1) * P_{OR} - \Delta t_j \tag{11}
\]
The global minimum jitter is then the smallest value which satisfies all local jitter conditions. As already said, \( \eta \) displays a pattern of distances between steps which repeats periodically every macro period. Therefore, it is sufficient to perform above calculation for one macro period. An algorithm can be found in [9].

### 4.4 Cyclic Task Dependencies

Tasks with multiple inputs allow to build cyclic dependencies. A typical application is a control loop, where one task represents the controller and the other task a model of the controlled system. A task graph with a cycle is shown in figure 12.

![Figure 12. Example of a cyclic dependency](image)

Tasks with multiple inputs in cycles are assumed to be AND-activated, and to produce one token at each output per execution. This implies that at least one initial token must be present inside the cycle to avoid deadlock [14], and that the number of tokens inside the cycles remains constant. Consequently, the period of the cycle-external event model determines the period of all cycle tasks. Finally, exactly one cycle-task with one cycle-external and one cycle-internal input is assumed to exist in a cycle. All other cycle-tasks only have cycle-internal inputs. These restrictions allow to concisely discuss the main issues resulting from functional cycles. A much more general discussion can be found in [10].

In section 4.2 it was established that the activation jitter of an AND-activated task is bounded by the largest input jitter. As was the case for cyclic scheduling dependencies (section 3.3.2), system analysis starts with an initial assumption about the cycle-internal jitter of the AND-activated task, since this value depends on the output jitter of that task, which have not been calculated yet. A conservative starting point is to initially assume zero internal jitter. Now analysis and event model propagation can be iterated around the cycle, hoping to find a fix-point.

However, if only one task along the cycle has a response time which is an interval, then after the first round of analysis and event model propagation the internal input jitter of the AND-activated task will be larger than the external input jitter. In the SymTA/S compositional performance analysis approach, this larger jitter will be propagated around the cycle again, resulting in an even larger jitter at the cycle-internal input of the AND-activated task (section 3.3.2). It is obvious that the jitter appears unbounded if calculated this way.

The problem boils down to the fact that event model propagation as presented so far captures neither correlations between the timing of events in different event streams, nor the fact that the number of tokens in a cycle is fixed. Therefore, the activation jitter for the AND-activated task is calculated very conservatively.
4.5 Analysis Idea

Cycle analysis requires detailed consideration of the possible phases between tokens arriving at the cycle-external and the cycle-internal inputs of the AND-activated task. The solution proposed in the following has the advantage to require only minor modifications to the feed-forward system-level analysis already supported by SymTA/S. The idea goes as follows:

Initially, the cycle-internal input is assumed to not increase the activation jitter of the AND-activated task. This allows to ‘cut’ the cycle-internal edge, rendering a feed-forward system which can be analyzed as explained in section 3.3.2. Then the time it takes a token to travel around the cycle is calculated, and the validity of the initial assumption is verified.

In the following, the idea is explained for cycles with one initial tokens. Assume an external periodic with jitter event model with period $P_{\text{ext}}$ and jitter $J_{\text{ext}}$. Let $t_{ff}^{\min}$ and $t_{ff}^{\max}$ be the minimum respectively maximum sum of worst-case response times of all tasks belonging to a cycle (the ’time around the cycle’) as obtained through analysis of the corresponding feed-forward system. Let us further assume that after analysis of the corresponding feed-forward system, $t_{ff}^{\max} \leq P_{\text{ext}}$.

At system startup, the first token arriving at the cycle-external input will immediately activate the AND-concatenated task together with the initial token already waiting at the cycle-internal input. No further activation of the AND-activated task is possible until the next token becomes available at the cycle-internal input of that task. If feed-forward analysis was valid, then this will take between $t_{ff}^{\min}$ and $t_{ff}^{\max}$ time units.

The maximum distance between two consecutive external tokens is $\delta_{\text{ext}}(2) = P_{\text{ext}} + J_{\text{ext}}$ (equation 4). From $t_{ff}^{\max} \leq P_{\text{ext}}$ follows that it is not possible that the 2nd external token arriving as late as possible after the 1st external token has to wait for an internal token.

The 3rd external token can arrive at most $\delta_{\text{ext}}(3) = 2 \cdot P_{\text{ext}} + J_{\text{ext}}$ after the 1st external token. Therefore, if both the 2nd and the 3rd external tokens arrive as late as possible, then the 3rd arrives $P_{\text{ext}}$ after the 2nd. From $t_{ff}^{\max} \leq P_{\text{ext}}$ follows that the 3rd external token arriving as late as possible after the 1st external token cannot wait for an internal token, even if the 2nd external token also arrived as late as possible. This argument can be extended to all further tokens. Thus, no external token arriving as late as possible has to wait for an internal token.

Activation of task $b$ also cannot happen earlier than the arrival of an external token. Therefore, the activating event model of task $b$ is conservatively captured by the external input event model (equation 12). This approach is therefore valid for a cycle with $M = 1$ initial token, for which $t_{ff}^{\max} \leq P_{\text{ext}}$.

$$P_{\text{act}} = P_{\text{ext}} \quad \text{;} \quad J_{\text{act}} = J_{\text{ext}}$$

For example, assume that in the system in Fig. 12 task $b$ is activated externally with $(P_{b,\text{ext}} = 4, J_{b,\text{ext}} = 3)$. Let us further assume that feed-forward analysis has determined the time around the cycle to be $[t_{ff}^{\min}, t_{ff}^{\max}] = [2, 3]$. I.e. each internal input event follows between $[2, 3]$ time units after the previous activating event. Fig. 13 shows a snapshot of a sequence of external, internal and activating events for task $b$ (numbers indicate corresponding input events and the resulting activating event). The first internal event is due to the initial token. As can be seen, activating event timing can be described by the same event model as external input event timing. If on the other hand analysis of the corresponding feed-forward system determines $t_{ff}^{\max} > P_{\text{ext}}$, then this statement is no longer true, since for example the 3rd internal event could occur later than the latest possible 3rd external event.
Figure 13. Possible event sequence for the cycle example. Gray boxes indicate jitter intervals during which an event can occur. Note that line 2 displays the possible timing of internal events depending on the previous activating event, while lines 1 and 3 display the possible timing of events independent of previous events.

In Fig. 13 it can also be seen that an early external token may have to wait for an internal token since two token arrivals at the cycle-internal input of task b cannot follow closer than $t_{ff}^{min}$, and thus

$$
\delta_{act}^{min}(2) = \begin{cases} 
\delta_{ext}^{min}(2) ; & t_{ff}^{min} \leq \delta_{ext}^{min}(2) \\
 t_{ff}^{min} ; & t_{ff}^{min} > \delta_{ext}^{min}(2)
\end{cases}
$$

Effectively, if $t_{ff}^{min} > \delta_{ext}^{min}(2)$, then the cycle acts like a $d_{min}$-EAF with $d_{min} = t_{ff}^{min}$ (section 3.4). This additional effect of the cycle does not require a new scheduling analysis, since the possible activation timing is only tightened. All possible event timing in the tighter model is already included in the wider model. Therefore, the results in equation 12 remain valid. However, it is worthwhile to perform scheduling analysis again with the tighter activating event model for the AND-concatenated task, since results will be more accurate.

In [10] it is shown that the approach presented in this section is also valid for a cycle with $M > 1$ initial tokens, for which $(M - 1) * P_{ext} < t_{ff}^{max} \leq M * P_{ext}$. In [10] it is also shown how to extend the approach to nested cycles. In SymTA/S, the feed-forward analysis is performed for every cycle, and the required number of initial tokens is calculated from $t_{ff}^{max}$. This number is then compared against the number of cycle-tokens specified by the user in the same manner as any other constraint is checked.

5. System contexts

Performance analysis as described so far can be unnecessarily pessimistic, because it ignores certain correlations between consecutive task activations or assumes a very pessimistic worst-case load distribution over time.

In SymTA/S, advanced performance analysis techniques taking correlations between successive computation or communication requests as well as correlated load distribution into account, have been added in order to yield tighter analysis bounds.
Cases where such correlations have a large impact on system timing are especially difficult to simulate and, hence, are an ideal target for formal performance analysis. Such correlations are called system contexts.

In Section 5.1, using an example of a hypothetical set-top box, the assumptions made by a typical performance analysis, called context blind analysis, is reviewed. Then, the analysis improvements that can be obtained when considering two different types of system contexts separately and also in combination are shown: intra event stream contexts, which consider correlations between successive computation or communication requests (section 5.2), and inter event stream contexts, which consider possible phases between events in different event streams (section 5.3). The combination of both system contexts is explained in section 5.4.

5.1 Context blind analysis

The SoC implementation of a hypothetical set-top box shown in figure 14 is used as an example throughout this section. The set-top box can process MPEG-2 video streams arriving from the RF-module ($rf_{video}$) and sent via the bus ($BUS$) to the TV ($tv$). In addition, a decryption unit ($DECRIPTION$) allows to decrypt encrypted video streams. The set-top box can additionally process IP traffic and download web-content via the bus ($ip$) to the hard-disk ($hd$).

![Figure 14. hypothetical set-top-box system](image)

The focus will be on worst-case response time calculation for the system bus. Assume static priority-based scheduling on the bus. The priorities are assigned as follows: $enc > dec > ip$. MPEG-2 Video frames are assumed to arrive periodically from the RF-module. The arrival period is normalized to 100. The core execution and communication times of the tasks are listed in table 1.

<table>
<thead>
<tr>
<th>task</th>
<th>CET</th>
</tr>
</thead>
<tbody>
<tr>
<td>$enc$</td>
<td>[10,30]</td>
</tr>
<tr>
<td>$dec$</td>
<td>[10,30]</td>
</tr>
<tr>
<td>$ip$</td>
<td>[50,50]</td>
</tr>
<tr>
<td>$decryption$</td>
<td>[40,40]</td>
</tr>
</tbody>
</table>

Table 1. Core execution times
The worst-case response time of $ip$, calculated by a context blind analysis, is 170. As can be seen in figure 15, even though a data dependency exists between $enc$ and $dec$, which may even out their simultaneous activation, a context blind analysis assumes that in the worst-case all communication tasks are activated at the same instant. Furthermore, even though MPEG-2 frames may have different sizes depending on their type, a context blind analysis assumes that every activation of $enc$ and $dec$ leads to a maximum transmission time of one MPEG-2 frame.

![Figure 15. Worst case response time calculation for $ip$ without contexts, using SymTA/S](image)

### 5.2 Intra Event Stream Context

Context-blind analysis assumes that in the worst-case, every scheduled task executes with its worst case execution time for each activation. In reality, different events often activate different behaviors of a computation task with different WCET, or different bus loads for a communication task. Therefore, a lower maximum load (and a higher minimum load) can be determined for a sequence of successive activations of a higher-priority task if the types of the activating events are considered. This in turn leads to a shorter calculated worst-case response time (and a longer best case response time) of lower-priority tasks. The correlation within a sequence of different activating events is called an *intra event stream context*. Mok, Chen and Baruah introduced this idea in [17] and showed promising results for MPEG-streams where the average load for a sequence of I-, P- and B-frames is much smaller than in a stream that consists only of large I-frames, which is assumed by a context-blind worst-case response time analysis. However, the periodic sequence of types of activating events was supposed to be completely known.

In reality, intra event stream contexts can be more complicated. If no complete information is available about the types of the activating events, it is no longer possible to apply Mok’s and Chen’s approach. Mok and Chen also do not clearly distinguish between different types of events on one hand, and different task behaviors, called *modes* [35], on the other. However, this distinction is crucial for subsystem integration and compositional performance analysis. Different types of events are a property of the sender, while modes are a property of the receiver. Both can be specified separately from each other and later correlated. Furthermore, it may be possible to propagate intra event stream contexts along a chain of tasks. It is then possible to also correlate the modes of consecutive tasks.

In SymTA/S, intra event stream contexts are extended by allowing minimum- and maximum-conditions for the occurrence of a certain type of events in a sequence of a certain length $n$, in order to capture partial information about an event stream. $n$ is an arbitrary integer value. A single worst-case and a single best-case sequence of events with length $n$ can be determined
from the available min- and max-conditions that can be used to calculate the worst- and best-case load due to any number of consecutive activations of the consumer task. In [12], the static-priority preemptive response-time calculation is extended to exploit this idea.

In the following, this approach is applied to the set-top box example. Suppose that the video stream sent from the RF to the bus, is encoded in one of several patterns of I-, P- and B-frames (IBBBBB, IBBPBB, IPBBBB...), or that several video streams are interleaved. Therefore, it is impossible to provide a fixed sequence of successive frame types in the video stream. However, it may be possible to determine min- and max-conditions for the occurrence of each frame type.

The communication times of tasks $\text{enc}$ and $\text{dec}$ depend on the received frame type. I-frames have the largest size and lead to the longest execution time, P-frames have the middle size and B-frames have the smallest size. Therefore, the mode corresponding to the transmission of an I-frame has the largest communication time and the mode corresponding to the transmission of a B-frame has the lowest communication time.

Having both intra event stream context information and modes of the consumer tasks, a weight-sorted worst case sequence of frame types with length $n$ can be determined. The reader interested in knowing the algorithm to exploit min- and max-conditions is referred to [12].

Now the worst case load produced on the bus can be determined for $l$ successive activations of $\text{enc}$ and $\text{dec}$. This is performed, by iterating through the weight-sorted sequence starting from the first event, adding up loads until the worst case load for $l$ activations has been calculated. If $l$ is bigger than $n$, the sequence length, the algorithm goes only through $l \mod n$ events and adds the resulting load to the load of the whole sequence multiplied by $l \div n$.

In figure 16, assuming that the worst case sequence of frame types with length 2 is: IP; and that the transmission time for an I-frame is 30 and for a P-frame is 20, the calculated worst case response time of $ip$, when considering the available intra event stream context information, is shown. As can be seen, for both tasks $\text{enc}$ and $\text{dec}$, the produced load on the bus due to a transmission of two successive MPEG-2 frames is smaller than in the context-blind case (see figure 15). This leads to a reduction of the calculated worst-case response time of $ip$: 150 instead of 170.

![Figure 16. Worst case response time calculation for $ip$ considering intra contexts](image)

5.3 Inter Event Stream Context

Context-blind analysis assumes that all scheduled tasks sharing a resource are independent and that in the worst-case all tasks are activated simultaneously. In reality, activating events are often time-correlated, which rules out simultaneous activation of
all tasks. This in turn may lead to a lower maximum number (and higher minimum number) of interrupts of a lower-priority task through higher-priority tasks, resulting in a shorter worst-case response time (and longer best-case response time) of the lower priority task. The correlation between time-correlated events in different event streams is called an *inter event stream context*.

Tindell introduced this idea for tasks scheduled by a static priority preemptive scheduler [30]. His work was later generalized by Palencia and Harbour [18]. Each set of time-correlated tasks is grouped into a so called *transaction*. Each transaction is activated by a periodic sequence of external events. Each task belonging to a transaction is activated when a relative time, called *offset*, elapses after the arrival of the external event.

To calculate the worst-case response time of a task, a worst-case scenario for its execution must be build. Tindell [30] showed that the worst-case interference of a transaction on the response time of a task occurs at the *critical instant* which correspond to the most delayed activation of a higher-priority task belonging to the transaction. The activation time of the analyzed task and all higher-priority tasks have to happen as soon as possible after the critical instant.

Since all activation times of all higher-priority tasks belonging to a transaction are candidates for the critical instant of the transaction, the worst-case response time of a lower-priority task has to be calculated for all possible combinations of all critical instants of all transactions that contain higher priority tasks, to find the absolute worst-case.

![Figure 17. Worst case response time calculation for ip considering inter contexts](image)

In the following, Tindell’s approach is applied to the set-top box example. Due to the data dependency between *enc*, decryption and *dec*, these tasks are time-correlated. The offset between the activations of *enc* and decryption corresponds to the execution time of *enc*. Based on this offset and the execution time of decryption, the offset between the activations of *enc* and *dec* can be calculated.

In order to show in isolation the analysis improvement due to inter event stream contexts, assume for now that all video-frames are I-frames. Figure 17 shows for the inter event stream context case the calculated worst case response time of *ip* due to interrupts by *enc* and *dec*. As can be seen, a gap exists between successive executions of *enc* and *dec*. Since *ip* executes during this gaps, one interrupt less of *ip* is calculated (in this case through *enc*). This leads to a reduction of the calculated worst-case response time of *ip*: 140 instead of 170.

In figure 18, analysis improvements with inter event stream context information in relation to the context-blind case are shown as a function of the offset between *enc* and *dec*, which is equal to the execution time of the decryption unit.

Curve *a* shows the reduction of the calculated worst-case response time of *dec*. Depending on the offset, *dec* is either partially (offset value less than 30), completely (offset value more than 70) or not interrupted at all by *enc* (offset value between 30
Figure 18. Improved worst-case response time calculation due to inter contexts

Curves b - g show the reduction in the calculated worst-case response time of ip for different IP traffic sizes. The reduction is visible in the curves as dips. If no gaps exists between two successive executions of enc and dec, no worst-case response time reduction of ip can be obtained (offset value less than 30 or more than 70). If a gap exists, then sometimes one interrupt less of ip can be calculated (either through enc or dec), or there is no gain at all (curves d and f). Since the absolute gain that can be obtained equals the smaller worst case execution time of enc and dec, the relative worst-case response time reduction is bigger for shorter IP-traffic.

An important observation is that inter event stream context analysis reveals the dramatic influence that a small local change, in the example the speed of the decryption unit reading data from the bus and writing the results back to the bus, can have on system-performance, in the example the worst-case transmission time of lower-priority IP traffic.

5.4 Combination of Contexts

Inter event stream contexts allow to calculate a tighter number of interrupts of a lower-priority task through higher-priority tasks. Intra event stream contexts allow to calculate a tighter load for a number of successive activations of a higher-priority task. The two types of contexts are orthogonal: the worst-case response time of a lower-priority task is reduced both because fewer high-priority task activations can interrupt its execution during a certain time interval, and because the time required to process a sequence of activations of each higher-priority task is reduced. Therefore, performance analysis can be further improved if it is possible to consider both types of contexts in combination. This is shown in figure 19 for the worst-case response time calculation of ip: 130 instead of 170.

In figure 20, analysis improvements considering both inter and intra event stream contexts in relation to the context-blind case are shown as a function of the offset between enc and dec. Curve a shows the reduction of the calculated worst-case response time of dec. Since dec is interrupted at most once by enc, and the worst-case load produced due to one activation
of $enc$ is the transmission time of one I-frame, no improvement is obtained through the context combination in comparison to curve a in figure 18.

Curves b - g show the reduction of the calculated worst-case response time of $ip$ for different IP traffic sizes. When comparing curves b and c (IP traffic sizes of 5 and 10) to curves b and c in figure 18, it can be seen that no improvement is obtained through the context combination. This is due to the fact that $ip$ is interrupted at most once by $enc$ and at most once by $dec$. Therefore, as in case a, the calculated worst-case load produced by the video streams is the same no matter whether the available intra event stream context information is considered or not.

Curve d shows that for an IP traffic size of 30, no improvements are obtained through the context combination in comparison to the context-blind case. This is due to the fact that for all offset-values, $ip$ is interrupted exactly once by $enc$ and exactly once by $dec$, and that the calculated worst-case load produced by the video streams due to one activation is the same no matter if intra event stream contexts are considered or not.

Curve e and f show that for IP traffic sizes of 50 and 70 improvements are obtained as a result of the context combination...
in comparison to both the intra and inter event stream context analysis. Since intra and inter event stream contexts are orthogonal, the reduction of the calculated worst-case response time of \textit{ip} due to the intra event stream context is constant for all offset values. Since no reduction due to inter event stream context can be obtained for an offset value of 0 (equivalent to the inter event stream context-blind case), the reduction shown in the curve for this offset value can only be a result of the intra event stream context. On the other hand, the additional reduction between the offset values 25 and 75 is obtained due to the inter event stream context.

Curve \textit{g} shows that for an IP traffic size of 90, even though the inter event stream context leads to an improvement (see curve \textit{g} in figure 18), the improvement due to the intra event stream context dominates, since no dip exists in the curve. I.e. no additional improvements are obtained due to the context combination in comparison to the intra event stream context case. This example shows that considering the combination of system contexts can yield considerably tighter performance analysis bounds compared to a context-blind analysis. Equally important, this example reveals the dramatic influence that a small local change can have on system-performance. Systematically identifying such system-level influences of local changes is especially difficult using simulation due to the large number of implementations that would have to be synthesized and executed. On the other hand, formal performance analysis can systematically and quickly identify such corner cases. All this results took a couple of milliseconds to compute using SymTA/S.

6. Design Space Exploration for System Optimization

This section gives an overview of the compositional design space exploration framework used in SymTA/S which is based on evolutionary optimization techniques. First system parameters which can be subject to optimization, i.e. the search space, are described. Then some examples of metrics expressing desired or undesired system properties used as optimization objectives in the exploration framework are presented. Afterwards we will explain how the search space can be defined and dynamically modified during exploration in order to allow the designer to guide the search process. Finally, the iterative design space exploration loop performed in SymTA/S is explained in detail.

6.1 Search Space

The entire system is seen as a set of independent \textit{chromosomes}, each representing a distinct subset of system parameters. A chromosome carries variation operators necessary for combination with other chromosomes of its type. Currently in SymTA/S, the standard operators mutation and crossover which are independently applied to the chromosomes, are used. The scope of a chromosome is arbitrary, it reaches from one single system parameter to the whole system. The search space and the optimization objectives can be multidimensional, which means that several system parameter can be explored simultaneously to optimize multiple objectives. Possible search parameter include:

- mapping of tasks onto different resources
- changing priorities on priority-scheduled resources
- changing time slot sizes and time slot order on TDMA or round robin scheduled resources
• changing the scheduling policy on a resource
• modifying resource speed
• traffic shaping

Traffic shaping is included into the search space because it increases the design space and allows to find solutions which are not possible without traffic modulation. This shall be shown with a small example.

Consider the task set in table 2 scheduled according to the static priority preemptive policy. All tasks are activated periodically except $T0$ which has a very large jitter leading to the simultaneous arrival of 3 activations in the worst case.

<table>
<thead>
<tr>
<th>Name</th>
<th>Activating Event Model</th>
<th>CET</th>
<th>Deadline</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T0$</td>
<td>$P(100) + J(200) + d(10)$</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$T1$</td>
<td>$P(100) + J(0) + d(0)$</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>$T2$</td>
<td>$P(100) + J(0) + d(0)$</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>$T3$</td>
<td>$P(100) + J(0) + d(0)$</td>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

Table 2. Simple task set

Two experiments are conducted. The first one with the original activating event models and the second one using a shaper at the input of $T0$ extending the minimum distance to 12. In the first experiment no priority assignment leading to a system fulfilling all constraints is found. However, in the second experiment the priority assignment $T0 > T1 > T2 > T3$ leading to a working system is found. The reason for this is that extending the minimum distance of successive activations of $T0$ relaxes the impact of the burst and leads to more freedom for the lower priority tasks to execute. This results in less preemption and thus earlier completion for $T1$, $T2$ and $T3$. Figures 21(a) and 21(b) visualize this effect by showing the worst-case scheduling scenarios for the priority assignment $T0 > T1 > T2 > T3$ with minimum distances 10 and 12.

Note that in the general case concerning distributed systems with complex performance dependencies, optimization through traffic shaping is not applicable in such a straight forward manner. Nevertheless, traffic shaping can broaden considerably the solution-space by restricting event streams, leading to increased freedom on cross-related event streams.

### 6.2 Optimization Objectives

The SymTA/S exploration framework is capable to perform a multi-objective optimization of several concurrent optimization objectives, leading usually to the discovery of several *pareto-optima*.

Pareto-optimal solutions represent a certain trade-off between two or more optimization objectives, leaving it to the designer to decide which solution to adopt. More precisely, given a set $V$ of $k$-dimensional vectors $v \in \mathbb{R}^k$. A vector $v \in V$ dominates a vector $w \in V$ if for all elements $0 \leq i < k$ we have $v_i \leq w_i$ and for at least one element $l$ we have $v_l < w_l$. A vector is called pareto-optimal if it is not dominated by any other vector in $V$.

Optimization objectives can be any kind of metric defined on desired or undesired properties of the considered system. Note that some metrics only make sense in combination with constraints. Each design alternative considered during the exploration process is associated with a fitness vector containing one entry for every concurrent optimization objective.
In the following some example optimization objectives used in the SymTA/S exploration framework will be introduced using the following notation:

- $R$ - maximum response time of a task or maximum end-to-end latency along a path
- $D$ - deadline (task or end-to-end)
- $\omega$ - constant weight $> 0$
- $k$ - number of tasks or number of constrained tasks/paths in the system

1. minimization of the (weighted) sum of completion times

$$\sum_{i=1}^{k} \omega_i \cdot R_i$$

2. minimization of the maximum lateness

$$\max(R_1 - D_1, \ldots, R_k - D_k)$$

**Figure 21. WC scheduling scenarios $T0 > T1 > T2 > T3$**
3. maximization of the minimum earliness
\[ \min(D_1 - R_1, \ldots, D_k - R_k) \]

4. minimization of the (weighted) average lateness
\[ \sum_{i=1}^{k} \omega_i \ast (R_i - D_i) \]

5. maximization of the (weighted) average earliness
\[ \sum_{i=1}^{k} \omega_i \ast (D_i - R_i) \]

6. minimization of end-to-end latencies

7. minimization of jitters

8. minimization of the sum of communication buffer sizes

The choice of the metric for optimization of a specific system is very important to obtain satisfying results. Example metrics 4 and 5, for instance, express the average timing behavior of a system with regard to its timing constraints. They might mislead an evolutionary algorithm and prevent it from finding system configurations fulfilling all timing constraints, since met deadlines compensate linearly for missed deadlines. For systems with hard real-time constraints, metrics with higher penalties for missed deadline and less rewards for met deadlines can be more appropriate, since they lead to a more likely rejection of system configurations violating hard deadline constraints. Following example metric penalizes violated deadlines in an exponential way and can be used to optimize the timing properties of a system with hard real-time constraints:
\[ \sum_{i=0}^{k} c_i^{R_i - D_i}, \ c_i > 1 \ constant \]

6.3 Defining the search space and controlling the exploration

The designer defines the current search space, by selecting and configuring a set of chromosomes representing the desired search space. System parameters not included inside the selected chromosomes remain immutable during the exploration. Figure 22 shows this principle.

The set of chromosomes representing the search space serves as blueprint for specific individuals (phenotypes) used during exploration. The variation operators (i.e. crossover and mutation) for these individuals are applied chromosome-wise.

The chromosomes are encoded and variated independently. There are two reasons why independent encoding and variation have been chosen. First, it is easier to establish a constructively correct encoding on a small subset of design decisions. Such an encoding scheme ensures that all chromosome values correspond to valid decisions such that any chromosome variation is constructively valid. This improves the optimization process as it greatly reduces the effort of checking a generated design for validity. It allows using the analysis engine of SymTA/S which requires correct design parameters to apply analysis (e.g. sum
of time slots no longer than the period, legal priority setting, etc.). Secondly, it is easy to add and remove design parameters to the optimization process, even dynamically, which is exploit in the exploration framework.

Chromosomes can be defined arbitrarily fine or coarse grain. This enables the designer to define the search space very precisely. She can limit certain parameters locally while giving others a more global scope. This way of defining the search space represents a compositional approach to optimization and allows to scale the search process. The designer can conduct several well directed exploration steps providing her insight into the system’s performance dependencies. Based on this knowledge she can then identify interesting design sub-spaces, worthy to be searched in-depth or even completely. An a priori global exploration does not permit such a flexibility and neglects the structure of the design space, giving the designer no possibility to modify and select the exploration strategy. In the worst-case, when the composition of the design space is unfavorable, this can lead to non-satisfying results with no possibility for the designer to intervene. In many approaches the only possibility for the designer in such a case consists in restarting the exploration, hoping for better results.

One important precondition for this approach to design space exploration is the dynamic configurability of the search space. The exploration framework allows the designer to redirect the exploration in a new direction without discarding already obtained results. She can for example downsize the search space by fixing parameters having the same values in (nearly) all obtained pareto-optimal solutions, or expand it with parameters not yet considered. Note that this methodology is more flexible than separate local parameter optimization and subsequent recombination.

6.4 Design Space Exploration Loop

Figure 23 shows the design space exploration loop performed in the exploration framework [8]. The Optimization Controller is the central element. It is connected to the Scheduling Analysis and to an Evolutionary Optimizer. The Scheduling Analysis checks the validity of a given system parameter set, that is represented by an individual, in the context of the overall heterogeneous system. The Evolutionary Optimizer is responsible for the problem-independent part of the optimization problem, i.e. elimination of individuals and selection of interesting individuals for variation. Currently, SPEA2 (Strength Pareto Evolutionary Algorithm 2) [36] and FEMO (Fair Evolutionary Multiobjective Optimizer) [13] are used for this part. They are coupled via PISA (Platform and Programming Language Independent Interface for Search Algorithms) [2].
Note that the selection and elimination strategy depends on the used multi-objective optimizer. For instance FEMO [13], eliminates all dominated individuals in every iteration and pursuits a fair sampling strategy, i.e. each parent participates in the creation of the same number of offsprings. This leads to a uniform search in the neighborhood of elitist individuals. The problem-specific part of the optimization problem is coded in the chromosomes and their variation operators.

Before the exploration loop can be started the designer has to select the desired search space (see section 6.1) and the optimization objectives (see section 6.2) she wants to optimize. The chromosomes representing the search space are included in the evolutionary optimization, while all other system parameters remain immutable. After the designer has selected the search space and the optimization task, SymTA/S is initialized with the immutable part of the system and the selected chromosomes are used as blueprints to create the initial population.

For each individual in the population the following is done:

- **Step 1.1**: The chromosomes of the considered individual are applied to the SymTA/S engine. This completes the system and it can be analyzed.

- **Step 1.2 + 1.3**: Each optimization objective requests the necessary system properties of the analyzed system to calculate its fitness value.

- **Step 1.4**: The fitness values are communicated to the Optimization Controller.

Once these 4 steps are performed for each individual inside the population the Optimization Controller sends a list of all individuals and their fitness values to the Evolutionary Optimizer (step 2). Based on the fitness values the Evolutionary Optimizer creates two lists: a list of individuals which are to be deleted and a list of individuals selected for variation and sends them back to the Optimization Controller (step 3). Based on the two lists the Optimization Controller then manipulates the population, i.e. he deletes the according individuals and creates new offsprings based on the individuals selected for variation and adds them to the population (step 4).
This completes the processing of one generation. The whole loop begins again with the new created population. After each iteration the designer can choose to modify the search space. This consists, like explained in section 6.3, in adding/removing chromosomes to/from the individuals. The reevaluation of the fitness values is performed automatically and the next exploration iteration is then started.

The performance of the search procedure in SymTA/S is affected by the search strategy of the optimizer, the coding of the chromosomes and their variation operations as well as the choice of the optimization objectives. As far as the optimizer is concerned, it is known that no general purpose optimization algorithm exists that is able to optimize effectively all kinds of problems [33].

7. Sensitivity analysis

Most analysis techniques known from literature give a pure Yes/No answer regarding the timing behavior of a specific system with respect to a set of timing constraints defined for that system. Usually the analyses consider a predefined set of input parameters and determine the response times, and thus, the schedulability of the system.

However, in a realistic system design process it is important to get more information with respect to the effects of parameter variations on system performance, as such variations are inevitable during implementation and integration. Capturing the bounds within which a parameter can be varied without violating the timing constraints offers more flexibility for the system designer and supports future changes. These bounds shows how sensitive the system or system parts are to system configuration changes.

Liu and Layland [16] defined a maximum load bound on a resource that guarantees the schedulability of that resource when applying a rate monotonic priority assignment scheme. The proposed algorithm is limited to specific system configurations: periodically activated tasks, tasks with deadlines at the end of their periods and tasks that do not share common resources (like semaphores) or that do not inter-communicate.

Later on, Lehoczky [15] extended this approach to systems with arbitrary task priority assignment. However, his approach does not go beyond the limitations mentioned above. Steve Vestal [32] proposed a fixed-priority sensitivity analysis for tasks with linear computation times and linear blocking time models. His approach is still limited to tasks with periodic activation patterns and deadlines equal to the period. Punnekkat [19] proposed an approach that uses a combination of a binary search algorithm and a slightly modified version of the response time schedulability tests proposed by Audsley and Tindell [1][31].

In the following is presented a brief overview about the sensitivity analysis algorithm and the analysis models and metrics used in SymTA/S. As already mentioned above, different approaches were proposed for the sensitivity analysis of different system parameters. However, these approaches can perform only single resource analysis as they are bounded by local constraints (tasks deadlines). Due to a fast increase of system complexity and heterogeneity, the current distributed systems usually have to satisfy global constraints rather than local ones. End-to-end deadlines or global buffer limits are an example of such constraints. Hence, the formal sensitivity analysis approaches used at resource level can not be transformed and applied at system level, as this implies huge effort and less flexibility.

The sensitivity analysis framework used in SymTA/S combines a binary search technique and the compositional analysis model implemented in SymTA/S. As described in section 3, SymTA/S couples the local scheduling analysis algorithms into
a global analysis model.

Since deadlines are the major constraints in real-time systems it makes sense to measure the sensitivity of paths latencies. As the latency of a path is determined by the response times of all tasks along that path, and the response time of a task directly depends on its core execution time, the following represent important metrics for the sensitivity analysis:

1. Maximum variation of the core execution time of a task without violating the system constraints or the system schedulability. If the system is not schedulable or constraints are violated then find the maximum value the task core execution time that leads to a conforming system.

2. Minimum speed of a resource. The decrease of a resource speed directly affects the core execution times of all tasks mapped on that resource but also reduces the energy required by that resource. If the system is currently not schedulable or constraints are violated then find the minimum resource speed that determines a conforming system.

**Variation of task execution/communication times** The search interval is determined by the current WCET value $t_{\text{core, max}}$ and the value corresponding to the maximum utilization bound of the resource holding the analyzed task. If the current utilization of resource $R$ is denoted by $R_{\text{load}}$ and the maximum utilization bound of resource $R$ is denoted by by $R_{\text{load, max}}$, then the search interval is determined by:

$$[t_{\text{core, max}}, t_{\text{core, max}} + P \times (R_{\text{load, max}} - R_{\text{load}})]$$

where $P$ represents the activation period in case of periodic tasks or the minimum inter-arrival time in case of sporadic tasks. If, for the current system configuration, the constraints are violated or the system is not schedulable then the search interval is determined by $[0; t_{\text{core, max}}]$.

The algorithm selects the interval middle value and verifies if the constraints are satisfied for the configuration obtained by replacing the task WCET value with the selected value. If yes, then the upper half of the interval becomes the new search interval, otherwise the lower half of the interval is searched. The algorithm iterates until the size of the search interval becomes smaller than a specific predefined value (abort condition).

**Variation of resource speed** The same algorithm is applied to find the minimum speed at which a resource can operate. If, for the current configuration, the constraints are satisfied and the system is schedulable then the search space is determined by $[R_{\text{speed, min}}, R_{\text{speed}}]$ where $R_{\text{speed}}$ is the current speed factor (usually 1) and $R_{\text{speed, min}}$ is the speed factor corresponding to the maximum resource utilization bound. Otherwise, the search space is $[R_{\text{speed}}, R_{\text{speed, max}}]$ where $R_{\text{speed, max}}$ is the speed factor corresponding to the maximum operational speed of that resource.

The ideal value for the maximum resource utilization bound is 100%. However, the experiments performed on different system models shown that, for utilization values above 98%, the run-time of the sensitivity analysis algorithm drastically increases. This is due to an increase of the analyzed period (busy period) in case of local analysis scheduling algorithms. Moreover, a resource load above 98% is not realistic in practice due to variations of the system clock frequency or other distorting elements.
In this section, using SymTA/S, the techniques from the previous sections are applied to analyze the performance of a system on chip example shown in figure 24.

The embedded system in figure 24 represents a hypothetical SoC consisting of a micro-controller (uC), a digital signal processor (DSP) and dedicated hardware (HW), all connected via an on-chip bus (Bus). DSP and uC are equipped with local memory. The HW acts as an interface to a physical system. It runs one task (sys_if) which issues actuator commands to the physical system and collects routine sensor readings. sys_if is controlled by task ctrl, which evaluates the sensor data and calculates the necessary actuator commands. ctrl is activated by a periodic timer (tmr) and by the arrival of new sensor data (AND-activation in a cycle). 2 initial tokens are assumed in the cycle.

The physical system is additionally monitored by 3 sensors (sens1 - sens3), which produce data sporadically as a reaction to irregular system events. This data is registered by an OR-activated monitor task (mon) on the uC, which decides how to update the control algorithm. This information is sent to task upd on the DSP, which updated parameters into shared memory. The DSP additionally executes a signal-processing task (fltr), which filters a stream of data arriving at input sig_in, and sends the processed data via output sig_out. All communication, except for shared-memory on the DSP, is carried out by communication tasks c1 - c5 over the on-chip Bus. Core execution times for each task are shown in Tab. 3.

The event models in table 4 are assumed at system inputs.

In order to function correctly, the system has to satisfy a set of path latency constraints (Tab. 5). Constraints 1 and 3 have been explicitly specified by the designer. The 2nd constraint implicitly follows from the fact that the cycle contains 2 initial tokens. Constraint 3 is defined for causally dependent tokens [34]. Additionally, a maximum jitter constraint is imposed at
computation task | C | communication task | C
--- | --- | --- | ---
mon | [10,12] | c1 | [8,8]
sys_if | [15,15] | c2 | [4,4]
fltr | [12,15] | c3 | [4,4]
upd | [5,5] | c4 | [4,4]
ctrl | [20,23] | c5 | [4,4]

Table 3. Core execution and communication times

<table>
<thead>
<tr>
<th>input</th>
<th>s/p</th>
<th>$P_{in}$</th>
<th>$J_{in}$</th>
<th>$d_{min.in}$</th>
</tr>
</thead>
</table>
sens1 | s | 1000 | 0 | 0 |
sens2 | s | 750 | 0 | 0 |
sens3 | s | 600 | 0 | 0 |
sig_in | p | 60 | 0 | 0 |
tmr | p | 70 | 0 | 0 |

Table 4. Event models at external system inputs.

<table>
<thead>
<tr>
<th>constraint #</th>
<th>path</th>
<th>maximum latency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>sens1, sens2, sens3 $\rightarrow$ upd</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>sig_in $\rightarrow$ sig_out</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>cycle (ctrl $\rightarrow$ ctrl)</td>
<td>140</td>
</tr>
</tbody>
</table>

Table 5. Path latency constraints

<table>
<thead>
<tr>
<th>constraint #</th>
<th>output</th>
<th>event model period</th>
<th>event model jitter</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>sig_out</td>
<td>$P_{sig.out} = 60$</td>
<td>$J_{sig.out, max} = 18$</td>
</tr>
</tbody>
</table>

Table 6. Output jitter constraint

8.1 Analysis

Static priority scheduling is used both on the DSP and the Bus. The priorities on the Bus respectively DSP are assigned as follows: $c1 > c2 > c3 > c4 > c5$ and $fltr > upd > ctrl$.

Performance analysis results were obtained using SymTA/S [7]. In the first step, SymTA/S performs OR-concatenation of the output event models of sens1 - sens3 and obtains the following sporadic activating event model for task mon:

$$P_{act} = P_{OR} = 250, J_{act} = J_{OR} = 500$$

The large jitter is due to the fact that input events happening at the same time lead to a burst of up to 3 activations (no correlation between sens1 - sens3 is assumed). Since task mon is the only task mapped onto $uC$, local scheduling analysis can now be performed for this resource, in order to calculate the minimum and maximum response times, as well as the output event model of task mon. The results of this analysis are shown in Tab. 7.
The worst-case response time of task $mon$ increases compared to its worst-case core execution time, since later activations in a burst have to wait for the completion of the previous activations. The output jitter increases by the difference between maximum and minimum core execution times compared to the activation jitter. The minimum distance between output events equals the minimum core execution time.

At this point, the rest of the system cannot be analyzed, because on every resource activating event models for at least one task are missing. SymTA/S therefore generates a conservative starting-point by propagating all output event models along all paths until an initial activating event model is available for each task. SymTA/S then checks that the system cannot be overloaded in the long term. This calculation requires only activation periods and worst-case core execution times and thus can be done before response-time calculation.

System-level analysis can now be performed by iterating local scheduling analysis and event model propagation. SymTA/S determines that task $ctrl$ belongs to a cycle, checks that AND-concatenation is selected, and then proceeds to analyze the corresponding feed-forward system. SymTA/S executes until a fix-point for the whole system has been reached, and then compares the calculated performance values against performance constraints.

Table 8 shows the calculated response times of the computation and communication tasks with and without taking into account inter contexts. As can be observed, the exploitation of context information leads to much tighter response time intervals in the given example. This in turn reduces the calculated worst-case values for the constrained parameters. Table 9 shows that, in contrast to the inter context blind analysis, all system constraints are satisfied when performance analysis takes inter context into account. In other words, a context blind analysis would have discarded a solution which is in reality valid.

| comp task |
| comp task |

| comp task |
| comp task |

| Table 7. Scheduling analysis results on uC |

| comp task |
| comp task |

| comp task |
| comp task |

| Table 8. Context blind and sensitive analysis |

| Table 9. Constraint values for context blind and sensitive analysis |
### Table 10. Pareto optimal solutions

<table>
<thead>
<tr>
<th>#</th>
<th>Bus tasks</th>
<th>DSP tasks</th>
<th>con. 1</th>
<th>con. 2</th>
<th>con. 3</th>
<th>con. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>c1, c2, c3, c4, c5</td>
<td>upd, fltr, ctrl</td>
<td>55</td>
<td>42</td>
<td>120</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>c1, c2, c4, c3, c5</td>
<td>upd, fltr, ctrl</td>
<td>59</td>
<td>42</td>
<td>112</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>c2, c1, c4, c5, c3</td>
<td>upd, fltr, ctrl</td>
<td>63</td>
<td>42</td>
<td>96</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>c1, c2, c3, c4, c5</td>
<td>fltr, upd, ctrl</td>
<td>70</td>
<td>27</td>
<td>120</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 11. Pareto optimal solutions: shaper at mon output

<table>
<thead>
<tr>
<th>#</th>
<th>Bus tasks</th>
<th>DSP tasks</th>
<th>con. 1</th>
<th>con. 2</th>
<th>con. 3</th>
<th>con. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>c2, c1, c3, c4, c5</td>
<td>upd, fltr, ctrl</td>
<td>59</td>
<td>42</td>
<td>120</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>c1, c2, c4, c3, c5</td>
<td>upd, fltr, ctrl</td>
<td>63</td>
<td>42</td>
<td>112</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>c3, c2, c1, c4, c5</td>
<td>fltr, upd, ctrl</td>
<td>64</td>
<td>35</td>
<td>120</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>c2, c1, c5, c4, c3</td>
<td>upd, fltr, ctrl</td>
<td>67</td>
<td>42</td>
<td>96</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>c2, c3, c1, c5, c4</td>
<td>fltr, upd, ctrl</td>
<td>68</td>
<td>31</td>
<td>134</td>
<td>7</td>
</tr>
</tbody>
</table>

#### 8.2 Optimizations

In this section, the architecture optimization of the system on chip example is shown. Optimization objectives are the four defined constraints. We try to minimize the latencies on paths 1-3 and the jitter at output \( \text{sig}_{out} \).

In the first experiment the search space consists of the priority assignments on the \textit{BUS} and the \textit{DSP}. Table 10 shows the existing pareto optimal solutions. In the first two columns, tasks are ordered by priority, highest priority on the left. In the last four columns, the actual value for all four constrained values is given. The best reached values for each constraint are emphasized.

As can be observed there are several possible solutions, each with its own advantages and disadvantages. We also observe that in each solution one constraint is only barely satisfied. A designer might want to find some alternative solutions where all constraints are fulfilled with a larger margin to the respective maximum values.

The search space is now extended by using a shaper at the output of task \textit{mon}. It is making sense to perform traffic shaping at this location, because the OR-activation of \textit{mon} can lead in the worst-case scenario to bursts at its output. That is, if all three \textit{sensors} trigger at the same time, \textit{mon} will send three packets over the \textit{BUS} with a distance of 10 time units, which is its minimum core execution time. This transient load peak affects the overall system performance in a negative way. A shaper is able to increase this minimum distance in order to weaken the global impact of the worst-case burst.

Table 11 shows pareto optimal solutions using a shaper at the output of \textit{mon} extending the minimum distance of successive events at the output of \textit{mon} to 12 time units, and thus weakening the global impact of the worst-case burst. The required buffer for this shaper is minimal, because at most one packet needs to be buffered at any time.

We observe that several new solutions are found. Not all best values for each constraint from the first attempt are reached, yet configurations 3 and 5 are interesting since they are more balanced regarding the constraints.
8.3 Sensitivity analysis

This section presents the results of the sensitivity analysis algorithms described in Section 7 applied to system configuration #2 shown in Table 10. Table 12 shows the current WCET, the maximum WCET allowed as well as the free WCET slack obtained for the particular configuration.

<table>
<thead>
<tr>
<th>task</th>
<th>current WCET</th>
<th>max WCET</th>
<th>slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>c2</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>c3</td>
<td>4</td>
<td>7.65</td>
<td>3.65</td>
</tr>
<tr>
<td>c4</td>
<td>4</td>
<td>10.65</td>
<td>6.65</td>
</tr>
<tr>
<td>c5</td>
<td>4</td>
<td>22.5</td>
<td>18.5</td>
</tr>
<tr>
<td>upd</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>fltr</td>
<td>15</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>ctrl</td>
<td>23</td>
<td>30</td>
<td>7</td>
</tr>
<tr>
<td>sys_if</td>
<td>15</td>
<td>36</td>
<td>21</td>
</tr>
<tr>
<td>mon</td>
<td>12</td>
<td>15.66</td>
<td>3.66</td>
</tr>
</tbody>
</table>

Table 12. Sensitivity analysis of tasks WCETs

The bar diagrams in Figure 25(a) shows the system flexibility with respect to variations of tasks WCETs. It can be easily stated that the tasks and channels along the filter path \((c1, c2, fltr)\) are very inflexible due to the jitter constraint defined at \(sig\_out\).

Table 13 presents the minimum resource speed factors that still guarantee the system meets all its constraints. A particular observation can be made considering the speed of \(BUS\). The results in Table 12 show that \(c1\) and \(c2\) are totally inflexible. However, from Table 13 it results that the \(DSP\) can be speed-down with maximum 26%. By applying the sensitivity analysis for \(c1\) and \(c2\) only the WCET has been modified, the BCET remaining constant. Contrary, changing the \(BUS\) speed both values, WCET and BCET, were changed. This led to a smaller jitter at channels output and to a higher flexibility for the \(BUS\) speed.

<table>
<thead>
<tr>
<th>resource</th>
<th>current factor</th>
<th>min factor</th>
<th>slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW</td>
<td>1</td>
<td>0.42</td>
<td>0.58</td>
</tr>
<tr>
<td>DSP</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>BUS</td>
<td>1</td>
<td>0.74</td>
<td>0.26</td>
</tr>
<tr>
<td>uC</td>
<td>1</td>
<td>0.77</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 13. Sensitivity analysis of resource speed factors

Figure 25(b) shows the results presented in Table 13. The timing constraints of filter-path \((c1, c2, fltr)\) and system-reactive-path \((mon, c3, upd)\) lead to rigid \(DSP\) properties with respect to later system changes.

9. Conclusion

The component integration step is critical in MpSoC design since it introduces complex component performance dependencies, many of them can not be fully overseen by anyone in a design team. Finding simulation patterns covering all corner cases
will soon become virtually impossible as MpSoCs grow in size and complexity, and performance verification is increasingly unreliable. In industry, there is an urgent need for systematic performance verification support in MpSoC design. The host of work in formal real-time analysis can be nicely applied to individual, local components or subsystems. However, the well established view on scheduling analysis has shown to be incompatible with the component integration style which is common practice in MpSoC design due to heavy component reuse. The recently adopted event stream view on component interactions represents a significant improvement for all kind of system performance related issues.

First, the stream model elegantly illustrates the consequences of a) resource sharing, and b) component integration, two of the main sources of complexity. This helps to identify previously unknown global performance dependencies, while tackling the scheduling problem itself locally where it can be overseen.

Secondly, the use of intuitive stream models such as periodic events, jitter, burst, and sporadic streams, allows to adopt existing local analysis and verification techniques. Essentially, SymTA/S provides automatic interfacing and adaptation among the most popular and practically used event stream models. In other words, SymTA/S is the enabling technology for the re-use of known local component design and verification techniques without compromising global analysis.

In this paper, the basic ideas underlying the SymTA/S technology is presented. SymTA/S has large variety of features that enable the analysis of complex embedded applications which can be found in practice. This includes multi-input tasks with complex activation functions, cyclic functional dependencies between tasks, systems with mutually exclusive execution modes, and correlated task execution (intra and inter contexts). These powerful concepts make SymTA/S a unique performance analysis tool that verifies end-to-end deadlines, buffer over-/underflows, and transient overloads. SymTA/S eliminates key performance pitfalls and systematically guides the designer to likely sources of constraint violations.

And the analysis with SymTA/S is extremely fast (10 seconds for the system in section 8, including optimization). The turn-around times are within seconds. This opens the door to all sorts of explorations, which is absolutely necessary for system optimization. SymTA/S uses genetic algorithms to automatically optimize systems with respect to multiple goals such as end-to-end latencies, cycles, buffer memory, and others. Exploration is also useful for sensitivity analysis in order to determine slack and other popular measures of flexibility. This is specifically useful in systems which might experience later
changes or modifications, a design scenario often found in industry. A large set of experiments demonstrate the application of SymTA/S and the usefulness of the results.

The SymTA/S technology was already applied in case studies in telecommunication, multimedia and automobile manufacturing projects. The cases had a very different focus. In one telecommunications project, a severe transient-fault system integration problem that not even prototyping could solve was resolved. In the multimedia case study, a complex two-stage dynamic memory scheduler was modeled and analysed to derive maximum response times for buffer sizing and priority assignment. In several automotive studies, it was shown how the technology enables a formal software certification procedure. The case studies have demonstrated the power and wide applicability of the event flow interfacing approach. The approach scales well to large, heterogeneous embedded systems including MpSoC. And the modularity allows to customize SymTA/S libraries to specific industrial needs.

The SymTA/S approach can be used as a serious alternative or supplement to performance simulation. The unique technology allows comprehensive system integration and provides much more reliable performance analysis results at far less computation time.

References


